# IMAGE FORMATION ALGORITHM FOR MISSILE BORNE MMW SAR WITH PHASE CODED WAVEFORM

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#### Abstract

The feasibility of the application with phase coded waveform to missile borne Millimetre Wave (MMW) Synthetic Aperture Radar (SAR) system was investigated. Based on theory analysis, the geometry model was simplified, the expression of the equivalent range and the Range-Doppler image formation algorithm were deduced simultaneously. The phase coded waveform is pulse compression signal, and it can depress the peak value power and provide large bandwidth, so the phase coded waveform is usually employed in SAR systems in order to acquire the low probability of intercept. The image results concentrated on the point array target and the area scene. Simulated results demonstrate that the SAR system with phase coded waveform has the ability to get the image of the target, and hence, the results validate the effectiveness of the image processing method.

# **1** Introduction

In order to acquire the high resolution in range direction, the signals SAR system transmitted should have large bandwidth; on the other hand, the wide band radar signals could enhance the concealment in modern radar. We can increase the bandwidth through frequency modulation or phase modulation. Phase modulation belongs to nonlinear modulation mode, the phase modulation function of which is discrete finite state, so the phase coded signal [8][1][5] is also called pulse compression signal. The outstanding advantage of the phase coded signal is that the radar peak value power can be depressed through utilizing the pulse compression method, and achieve the low probability of intercept. However, once the echo signals are unmatched with the filter, the filer is invalid too. Therefore, the phase coded signal is sensitive to the Doppler information <sup>[2][6]</sup>, in practice; the Doppler bound should be limited. Due to the flexibility of the phase coded mode, the high performance radar usually use the phase coded signal to realize the wave fast change. At present, the phase coded mode is utilized in pulse Doppler radar usually, the concept of applying phase coded signal in SAR system is a novel idea, the analysis of phase coded signal in SAR system opens an important domain in modern radar.

Missile borne SAR has been applied widely into air combat in virtue of far range. High azimuth resolution performance can benefit from transmission of MMW signals in SAR system

and the difficulty of the image formation may be decreased correspondingly. There is some literature on the work about MMW SAR. Reference <sup>[3]</sup> analyses the characteristic of air-toair missile borne SAR imaging, and a modified extended chirp scaling (ECS) algorithm is used in high forward-looking squint image processing. And an advanced simulation system for MMW imaging radar seeker onboard air-to-air is studied in reference<sup>[7]</sup>. The reference about application of phase coded waveforms in MMW SAR system is seldom reported, and therefore, it is helpful to pay attention to the SAR system with phase coded signal. Phase coded waveforms are applied in MMW SAR in this paper in order to improve the ability of anti-jamming. Section II introduce the geometry model of the SAR and section III provides the SAR echo model and RD image formation algorithm with phase coded waveform. Section IV presents the simulation results of point array and the aeroplane. The conclusion is discussed in section V.

#### **2** Geometry model

On condition that the SAR platform is a missile, figure 1<sup>[4]</sup> illustrates the relation between platform and the target. Overlooking the impact of earth rotation, Suppose the ground is flat and the target is immobile relatively. The velocity of the SAR is V, X direction is the flight direction, H is the altitude of the SAR,  $\theta$  is the squint angle (the angle between beam center and the flight direction).



Figure 1. Geometry relation between radar and target

At t=0, the slant range between the beam center and the target T is  $R_0$ , then the relative distance at time t is as follows:

$$R(t) = \sqrt{\left(\left(R_0 \cdot \cos\theta - vt\right)^2 + \left(R_0 \cdot \sin\theta\right)^2\right)}$$
  
=  $R_0^2 - 2vR_0 \cos\theta \cdot t + v^2 \cdot t^2$  (1)

We utilize the Taylor series to spread the R(t) at t=0:

$$R(0) = R_{0}$$

$$R'(t) = \frac{v^{2} \cdot t - vR_{0} \cos \theta}{\sqrt{R_{0}^{2} - 2vR_{0} \cos \theta \cdot t + v^{2} \cdot t^{2}}}$$

$$R'(0) = -v \cos \theta$$

$$R''(t) = \frac{v^{2}R_{0}^{2} \sin^{2} \theta}{(R_{0}^{2} - 2vR_{0} \cos \theta \cdot t + v^{2} \cdot t^{2})^{3/2}}$$

$$R''(0) = \frac{v^{2} \sin^{2} \theta}{R_{0}}$$

Then if ignoring the items which is much higher than three, we can get the R(t)

$$R(t) = R(0) + R'(0) \cdot t + \frac{R(0)}{2} \cdot t^{2}$$

$$= R_{0} - v \cos \theta \cdot t + \frac{v^{2}}{2 \cdot R_{0}} \sin^{2} \theta \cdot t^{2}$$
(2)

# 3 SAR echo model with phase coded waveform

The modulation function of linear frequency wave is continuous, which belongs to the continuous signal; while the modulation function of the phase coded signal is discrete, which can be described as discrete pulse compression signal. Owing to the phase coded signal adopt the pseudo-random sequence, so the phase coded signal is also called pseudorandom coded signal.

## 3.1 SAR echo analysis

The expression of the phase coded signal is  $\phi(t) = a(t)e^{j\phi(t)}e^{j2\pi f_c t}$ , the plural envelope is  $u(t) = a(t)e^{j\phi(t)}$ , where  $\phi(t)$  is the phase modulation function, for bi-phase coded signal,  $\phi(t)$  has only two value 0 and 1, which can be denoted as  $\{c_k = e^{j\phi_k} = +1, -1\}$ , where  $\phi_k = \{0, \pi\}$ . Let the envelope of the bi-phase coded signal is rectangle.

$$a(t) = \begin{cases} 1/\sqrt{N\tau_1} & 0 < t < T = N\tau_1 \\ 0 & \text{others} \end{cases}$$
(3)

The plural envelope of the bi-phase coded signal is:

$$u(t) = \begin{cases} \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} c_k v(t - k\tau_1) & 0 < t < T\\ 0 & \text{others} \end{cases}$$
(4)

In this paper, we use baker code with the length of 13 as the phase coded signal, and the waveform chart is listed in figure 2. Figure 3 shows the ambiguity function of the barker sequence. From figure 3, we can know that the shape of the ambiguity function which the barker code showed is similar

to a thumbtack. And the high resolution that the barker code processed is demonstrated.



Figure2 Waveform of the barker code



Figure3 Ambiguity function of the barker code

Suppose S is the transmitted signal, the pulse width is  $T_p$ , the sub-pulse width is  $\tau_1$ ,  $T_p = N \cdot \tau_1$ , where N is the length of the phase coded signal

$$S(\tau) = rect(\frac{\tau}{T_p}) \cdot \exp(j(2\pi f_c \tau + \phi(\tau)))$$
(5)

Where,  $f_c$  is the carrier wave frequency of S,  $\phi(\tau)$  denotes the phase coded function, and

$$rect(\frac{\tau}{T_p}) = \begin{cases} 1 & (-T_p / 2 \le \tau \le T_p / 2) \\ 0 & (others) \end{cases}$$
(6)

The echo signal of the target T is:

$$S_{r}(t,\tau) = \sigma \cdot rect(\frac{t}{T_{s}}) \cdot \exp[j2\pi f_{c}(\tau - \frac{2R(t)}{C})]$$

$$\cdot rect(\frac{\tau - \frac{2R(t)}{C}}{T_{p}}) \cdot \exp[j \cdot \phi(\tau - \frac{2R(t)}{C})]$$
(7)

Where  $\sigma$  is the back scatter coefficient,  $T_s$  is the synthetic aperture time, the target is irradiated during the time  $T_s$ , then eliminates the phase produced by  $f_c$ , the echo signal is:

$$S_{r}(t,\tau) = \sigma \cdot rect(\frac{t}{T_{s}}) \cdot \exp[-j\frac{4\pi}{R}R(t)]$$

$$\cdot rect(\frac{\tau - \frac{2R(t)}{C}}{T_{p}}) \cdot \exp[j \cdot \phi(\tau - \frac{2R(t)}{C})]$$
(8)

## **3.2 Compression process**

Due to the transmitted signal is the phase coded signal, so the performance of the range direction signal dissatisfied the property of the linear frequency wave, therefore, the matching filter should have the coherence with the phase coded signal in order to meet the pulse compression correctly.

Based on the echo signal model, we process the echo data through RD algorithm in order to gain the image of the target. The flowchart of the RD algorithm is described in figure 4.



Figure 4 Block diagram of RD algorithm

The reference functions of the range direction and the azimuth direction are:

$$h_r^{-1}(t) = \exp\{j[2\pi f_0 t - \phi(t)]\}$$
(9)

$$h_a^{-1}(t) = \exp\{j2\pi(f_{d_0}t + \frac{1}{2}f_{r_0}t^2)\}$$
(10)

Where  $f_0$  is the central frequency of the instantaneous frequency, and  $\phi(t)$  is the corresponding function to the transmitted phase coded signals.

## 4 Simulation

Table 1 illustrates the parameters in the simulation, we utilize the barker sequence as the transmitted phase coded signal, the length of the barker sequence is 13, the sample number in azimuth direction is 1024 and 512 in range direction.

Parameters	Value
Wavelength (m)	0.008
Sub-pulse width ( $^{\mu}$ s)	0.65
Wideband (MHz)	20
Antenna length (m)	2
Pulse Repetition Frequency (Hz)	3625
Sampling frequency (MHz)	40
Velocity (m/s)	1200
Slant range distance (km)	10

#### Table 1. Main simulation parameters

Next, the echo data will be processed. In the experiment, the  $3 \times 3$  point array and an aeroplane are simulated. Figure 5 shows the result after range direction compression and figure 6 shows the result after azimuth direction compression, figure 7 and figure 9 are the image results.



Figure 5 Compression result after range direction



Figure 6 Compression result after azimuth direction

The image results are:



Figure 7 Image result of Point array target

Another simulation target was an aeroplane, the simulation parameters are the same as that of point target. Figure 8 shows the prototype an aeroplane and the imaging result is shown in figure 9.



#### Figure 8 Prototype of an aeroplane



Figure 9 Image result of the aeroplane

The image results above show that the SAR system based on the phase coded signal is feasible, at the same time, it is proved that the RD algorithm is valid in the phase coded signal SAR system.

## 5 Conclusion

This paper tends to investigate the feasibility of the application of the MMW SAR system with phase coded signal in theory. The geometry model and the slant range model were deduced, the SAR image processing method with phase coded signal was also put forward. The image results of the point array targets show that the algorithm satisfied the theory value and demonstrated the validity of the image processing method. In the end, the aeroplane imaging results reflect the potential advantage of the MMW SAR system with phase coded signal. This paper opens a new channel in the choose of the transmitted signals in SAR systems, there are still many key technologies in theory and in practice to be researched further, such as mobile target identification and anti-interference and so on.

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