# Multiple-Input Single-Output Synthetic Aperture Radar and Space-Time Adaptive Processing

# A Thesis

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By

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### ABSTRACT

This thesis investigates the plausibility of implementing a multiple-input singleoutput (MISO) synthetic aperture radar (SAR) system for space-time adaptive processing (STAP) with a limited data rate requirement of a single receiver. A MISO-SAR system could provide processing flexibility to radar systems such as the Gotcha radar system developed at the Air Force Research Laboratory. Gotcha is an airborne wide-beam multi-mode radar system used to cover a large area for surveillance. In order to apply multiple algorithms to a large amount of data in real time, the data is downlinked to a supercomputer on the ground. STAP is an adaptive filtering technique which can be used for improved detection of slow moving targets in the presence of clutter. However, STAP is typically implemented using an array of receiving elements, which significantly increases the data rate for downlinking to the ground. While MISO systems are common in communications applications, it is not a common radar system design approach. The MISO system requires additional waveform design considerations in order to obtain orthogonal transmit waveforms. However, the MISO system provides the additional degrees of freedom needed to apply STAP while maintaining a single receiver data rate.

This is dedicated to my family who's support has been invaluable.

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## CHAPTER 1

#### MOTIVATION

Multiple-input single-output (MISO) systems are commonly used for communications applications. However, radar has traditionally employed single-input singleoutput (SISO) and single-input multiple-output (SIMO) systems. Multiple-input multiple-output (MIMO) radar has been researched more recently. The advantage of a MISO radar system is the ability to obtain the additional angular information obtained by SIMO radar systems, while keeping the received data rate as low as the SISO radar system. The MISO system, similar to a MIMO system, requires the use of orthogonal waveforms and additional processing of the phase history.

The multiple-input single-output system was motivated by the goals of the Gotcha Radar Exploitation Program (GREP) developed at the Sensors Directorate of Air Force Research Laboratory. The Gotcha program is a circular synthetic aperture radar (C-SAR) which currently employs a SIMO system to image a large ground scene of an urban environment from a 45<sup>o</sup> grazing angle [3]. The radar system collects phase history data and processes the data digitally using a supercomputer on the ground in real-time. The high data rate for a large scene size requires a single receiver system in order to process in real-time using available down-link hardware. Besides imaging, another goal of the system is to track moving targets. Currently, tracking moving targets using change detection and path predictions algorithms is implemented . In order to create a more robust tracking algorithm using STAP, this thesis will explore the possibility of implementing a MISO-SAR system.

#### CHAPTER 2

#### TECHNICAL BACKGROUND

This chapter describes standard synthetic aperture radar (SAR) and space-time adaptive processing (STAP). The background information is intended to assist in the understanding of the derivations and demonstrations in Chapter 3.

#### 2.1 Synthetic Aperture Radar

Synthetic aperture radar (SAR) exploits the motion of a moving platform to create a synthetic antenna array, which improves cross-range resolution. Compared to aerial photographs, SAR imaging has the advantage of producing images in bad weather and at night. A top-level block diagram illustrating the typical single-input single-output SAR system model described in this section is shown in Figure 2.1.

## 2.1.1 Synthetic Aperture Radar Equations

Since the main goal of synthetic aperture radar is to produce high resolution images, the waveforms used are high bandwidth. As discussed in [7] and [8], the typical SAR uses linear frequency modulation (LFM) waveforms and pulse compression.

The range resolution is

$$\delta_R = \frac{c}{2BW},\tag{2.1}$$



Figure 2.1: Typical SAR System Model.

where BW is the bandwidth of the linear FM chirp.

The cross-range resolution of a real aperture radar is

$$\delta_{CR} = 2R\sin(\frac{\theta_{AZ}}{2}) \approx R\theta_{3dB}, \qquad (2.2)$$

where  $\theta_{AZ}$  is the 3dB beamwidth of the antenna in the azimuth plane. Since  $\theta_{AZ}$  is proportional to  $\frac{\lambda}{D_{AZ}}$ , where  $\lambda$  is the wavelength and  $D_{AZ}$  is the dimension of the antenna in the azimuth plane, the cross range resolution for SAR is

$$\delta_{CR} \approx \frac{R\lambda}{D_{AZ}}.$$
(2.3)

By taking advantage of the platform motion, as discussed in [7], the effective antenna dimension in azimuth can be defined as

$$D_{SAR} = vT_a, \tag{2.4}$$

where  $T_a$  is the integration time of the aperture. Therefore, the cross-range resolution can be reduced to

$$\delta_{CR} \approx \frac{R\lambda}{vT_a}.\tag{2.5}$$

The general rule for the possible cross-range resolution which can be achieved by SAR with  $T_a = \frac{R\theta_{AZ}}{v}$  is

$$\delta_{CR} = \frac{D_{AZ}}{2}.\tag{2.6}$$

The ability to substantially improve cross-range resolution is what makes SAR so useful.

In order to illustrate typical SAR processing, a signal model will be discussed. Let s(t) represent a complex-valued transmit waveform. Then  $r_g(t) = \sigma_g s(t - \Delta t_g)$ represents the received signal from the  $g^{th}$  scatterer at a time delay of  $\Delta t_g$  with a radar cross section (RCS)  $\sigma_g$ .

For a standard linear FM pulse waveform,

$$s(t) = \exp\{j2\pi(\frac{\alpha}{2}t + f_c)t\}, \quad -\frac{T}{2} < t < \frac{T}{2}.$$
(2.7)

where  $\alpha$  is the chirp sweep rate,  $f_c$  is the center frequency, and T is the pulse duration.

Therefore, the received signals from the  $g^{th}$  scatterer at the two-way time delay,  $\Delta t_q$ , is defined as

$$r_g(t) = \sigma_g s(t - \Delta t_g)$$

$$= \sigma_g \exp\{j2\pi(\frac{\alpha}{2}(t - \Delta t_g) + f_c)(t - \Delta t_g)\}, \quad \Delta t_g - \frac{T}{2} < t < \Delta t_g + \frac{T}{2}(2.9)$$

For simplicity, the radar cross section is assumed to be a complex amplitude without frequency or angle dependence for a point scatterer radiating isotropically.

Next, consider the sampled data forms of the transmit and received pulses for sampling period  $T_s$ . For the transmit pulses

$$s[n] = s(nT_s), \qquad n = -\frac{N}{2}, ..., \frac{N}{2},$$
(2.10)

where the number of samples per transmitted pulse is

$$N = floor(\frac{T}{T_s}). \tag{2.11}$$

Similarly, the number of samples per pulse on receive is given by

$$L = \frac{t_{stop} - t_{start}}{T_s} = \frac{(t_{far} + T/2) - (t_{near} - T/2)}{T_s},$$
(2.12)

where  $t_{start}$ ,  $t_{stop}$ ,  $t_{near}$ , and  $t_{far}$  are the time to start receiving returns from the scene, the time to stop receiving returns from the scene, the two-way time delay to near scene, and the two-way time delay to far scene, respectively, as illustrated in Figure 2.2. Therefore, the receive indices are  $l = -\frac{L}{2}, ..., \frac{L}{2}$ . For a change of variables to let l be zero at scene center, let

$$t = nT_s - T/2 = lT_s + t_{start},$$
(2.13)

which yields receive samples

$$r_g[l] = r_g(lT_s) = \sigma_g s(lT_s - \Delta t_g).$$
(2.14)

The phase history over one slow time pulse, n, is a summation over all reflectors,

$$r_n[l] = \sum_{g=0}^G r_g[l].$$
 (2.15)

where G is the number of point scatterers. The total phase history over all slow time pulses will be saved as a matrix of column vectors

$$PH = [r_1[l], r_2[l], ..., r_N[l]],$$
(2.16)

where n=1, 2,...,N represent the slow time indices.

Assuming linear FM waveforms, pulse compression is used to produce a range compressed phase history. This can be down using matched filtering or a technique



Figure 2.2: Received chirps from one transmitted chirp.

called stretch processing. The matched filter approach is intuitive, but requires faster sampling than stretch processing,

$$r[l] = s^*[-n] \sim PH[n], \tag{2.17}$$

where  $s^*$  denotes the complex conjugate of s and ~ denotes convolution.

Stretch processing includes LFM de-ramping, which will be discussed for practical implementation due to the decreased sampling frequency requirement. A de-ramp reference signal is applied to the received signals as a matched filter. This reference signal is denoted as

$$w[l] = s(lT_s + t_{start} - \Delta t_{center}), \qquad (2.18)$$

where  $\Delta t_{center} = 2R_{center}/c$  and  $R_{center}$  is the range to scene center. Note that this reference signal is over the entire receive period for one slow time pulse. For a single received pulse  $r_g[l] = \sigma_g s(lT_s + t_{start} - \Delta t_g)$ , the de-ramped receive signal is

$$d_g[l] = r_g[l]w^*[l]. (2.19)$$

#### 2.1.2 Backprojection

Backprojection is one imaging technique to form focused images from range compressed phase history [4]. The range resolution for the imaging is achieved by pulse compression across fast time as previously described in Equation 2.1. In order to achieve the same resolution in cross-range, the platform motion is used to process the returns across slow time pulses. Backprojection can be thought of as a method of applying cross-range compression for producing an image.

The backprojection algorithm has been derived in [4] with example Matlab code accompanying the paper for efficient understanding of the backprojection process.

#### 2.2 Space-Time Adaptive Processing

Space-time adaptive processing (STAP) is the technique of simultaneously processing the temporal and spatial characteristics of a signal. STAP is mostly used in radar applications to improve moving target detection in the presence of clutter, and even jammers. This chapter discusses the space-time adaptive processing method for a standard SIMO array radar system as described in [10] and [7]. However, the derivation has also been adapted for the more complicated MIMO system [2]. The STAP derivation and the modifications to its application are relatively extensive. The focus of this section is to provide sufficient background for the purposes of this thesis. A thorough explanation of the STAP algorithm can be found in the previously mentioned references.

Although STAP can be implemented to counteract jamming, the main focus of this discussion relates to the ability to to improve the detection of moving targets in the presence of ground clutter. There have been several approaches to STAP researched over the past couple decades [10]. This section will focus on the general form of STAP, referred to as fully adaptive STAP. Fully adaptive STAP is typically impractical for real-world implementation due to limited data and computational complexity [10]. However, understanding fully adaptive STAP is crucial to the consideration of modified STAP algorithms.

Figure 2.3 illustrates a coherent processing interval, similar to the representation seen in [10] and [7]. One should note that M represents the number of elements and N represents the number of pulses to avoid confusion with previous notation. The cube shown here represents the data used in the STAP algorithm. The M rows represent the M spatial antenna elements and the N columns represent the N slow time pulses



Figure 2.3: Coherent Processing Interval (CPI).

of the radar system. The third dimension represents the L range compressed fast time bins. In the general STAP algorithm, the M elements are assumed to have identical antenna parameters. The STAP algorithm considers each l = 1, ..., L range slice separately. From Figure 2.3, each range slice consists of a  $M \times N$  matrix. A space-time data snapshot, denoted as Y, is a  $NM \times 1$  vector of a particular range bin, l, formed by stacking the columns of the  $M \times N$  matrix. The top-level STAP algorithm described in this section is illustrated by Figure 2.4.



Figure 2.4: Typical STAP System Model.



Figure 2. (a) Platform geometry. (b) Top view

Figure 2.5: Spatial geometry [10].

### 2.2.1 Spatial Steering Vector

Spatial steering is achieved by using the M multiple phase centers. This discussion is based on a SIMO system. One antenna is transmitting and a uniform linear array of elements are receiving. For m = 0, 1, ..., M - 1 elements, the return of a single fast time has a phase shift associated with the angle of arrival (AOA) as seen in Figure 2.5. The  $n^{th}$  pulse snapshot of the CPI will be denoted as

$$y_n(t) = A [\exp\{j2\pi f_c(t - md\cos\theta\sin\phi/c) + \Psi_0\}]^T,$$
(2.20)

where  $\Psi$  is the arbitrary initial phase of the m = 0 element, d is the element spacing,  $\theta$  is the elevation angle, and  $\phi$  is the azimuth angle. The amplitude, A, is ignored throughout the rest of the derivation since the phase change is the only concern. However, this amplitude is the component detected after the steering vectors are applied. Note that this derivation is based on a constant frequency waveform design and ignores bandwidth. The discrete returns are then defined as

$$y[n] = y_n(t_0) = [\exp\{j2\pi md\cos\theta\sin\phi/\lambda\}]^T, \qquad (2.21)$$

where m = 0...M - 1.

By defining the spatial frequency as  $f_s = d \cos \theta \sin \phi / \lambda$ , a spatial snapshot is then defined across these M spatial channels as

$$\mathbf{y_n} = [y_n[0], y_n[1], \dots, y_n[M-1]]^T$$
(2.22)

$$= [1, e^{j2\pi d\cos\theta\sin\phi/\lambda}, ..., e^{j2\pi (M-1)d\cos\theta\sin\phi/\lambda}]^T$$
(2.23)

$$= [1, e^{j2\pi f_s}, \dots, e^{j2\pi (M-1)f_s}]^T.$$
(2.24)

Therefore the spatial steering vector is denoted as

$$\mathbf{a}_{\mathbf{s}}(f_s) = [1, e^{j2\pi f_s}, ..., e^{j2\pi (M-1)f_s}]^T$$
(2.25)

or equivalently,

$$\mathbf{a}_{\mathbf{s}}(\theta,\phi) = [1, e^{j2\pi d\cos\theta\sin\phi/\lambda}, ..., e^{j2\pi (M-1)d\cos\theta\sin\phi/\lambda}]^T$$
(2.26)

$$\mathbf{a}_{\mathbf{s}}(\theta_c) = [1, e^{j2\pi d\cos\theta_c/\lambda}, ..., e^{j2\pi (M-1)d\cos\theta_c/\lambda}]^T, \qquad (2.27)$$



Figure 2.6: AOA denoted as  $\theta_c$  [5].

where  $\cos \theta_c = \cos \theta \sin \phi$  is the angle of arrival (AOA) as shown in Figure 2.6

The spatial steering vector is a basis vector of the discrete Fourier transform (DFT).

#### 2.2.2 Temporal Steering Vector

The temporal variation across slow time allows for Doppler processing of the phase history. The Doppler shift is related to the AOA as follows

$$f_d = \frac{2v}{\lambda},\tag{2.28}$$

where v is the radial velocity. The Doppler frequency can be normalized by the pulse repetition frequency (PRF),  $f_p$  and is denoted as

$$\tilde{f}_d = \frac{2vT_p}{\lambda} = \frac{f_d}{f_p}.$$
(2.29)

Therefore, the temporal steering vector is typically defined as

$$\mathbf{a}_{\mathbf{t}}(\tilde{f}_d) = [1, e^{j2\pi \tilde{f}_d}, ..., e^{j2\pi (N-1)\tilde{f}_d}]^T.$$
(2.30)

The temporal steering vector is also a basis vector of the DFT.

### 2.2.3 Space-Time Steering Vector

In order to represent the temporal and spatial phases, the Kronecker product of the  $M \times 1$  spatial steering vector and the  $N \times 1$  temporal steering vector is taken to produce the  $MN \times 1$  space-time steering vector. The space-time steering vector is defined as

$$\mathbf{a}_{\mathbf{s},\mathbf{t}} = \mathbf{a}_{\mathbf{t}} \otimes \mathbf{a}_{\mathbf{s}},\tag{2.31}$$

where  $\otimes$  represents the Kronecker matrix product. Therefore, the space-time steering vector is the unit amplitude system response of a target with spatial frequency  $f_s$  and normalized Doppler  $\tilde{f}_d$ .

### 2.2.4 Noise, Clutter, and Jamming

The interference present in a radar system can be separated into three types; noise, clutter, and jamming. Each of these forms of interference is represented by the following covariance matrices, as described in [7] and [10].

The simple model of internal receiver noise is

$$\mathbf{R}_{\mathbf{n}} = \sigma^2 \mathbf{I}_{\mathbf{MN}},\tag{2.32}$$

where  $\sigma^2$  is the noise power of the receiver.

Although not the focus of this thesis, the model typically used to represent jamming signals is

$$\mathbf{R}_{\mathbf{J}} = \mathbf{I}_{\mathbf{M}} \otimes \mathbf{M}_{\mathbf{J}},\tag{2.33}$$

where  $M_J$  is the spatial covariance of the jammer.

The focus of STAP in this thesis is to reduce clutter interference. The clutter model derivation is very extensive and relies on several system and geometric variables. The clutter model, derived in [10], is

$$\mathbf{R}_{\mathbf{c}} = \sigma_c^2 \sum_{i=1}^{N_r} \sum_{k=1}^{N_c} \zeta_{ik} (\mathbf{a}_{\mathbf{t}_{ik}} \mathbf{a}_{\mathbf{t}_{ik}}^{\ H}) \otimes (\mathbf{a}_{\mathbf{s}_{ik}} \mathbf{a}_{\mathbf{s}_{ik}}^{\ H}), \qquad (2.34)$$

where the  $ik^{th}$  clutter patch is defined by azimuth  $\phi_k$ , elevation  $\theta_i$ , clutter-to-noise ratio  $\zeta_{ik}$ , and expected clutter power  $\sigma_c^2$ .

Therefore, the total interference is represented by

$$\mathbf{R}_{\mathbf{I}} = \mathbf{R}_{\mathbf{n}} + \mathbf{R}_{\mathbf{J}} + \mathbf{R}_{\mathbf{c}}.$$
 (2.35)

As described in [10], the clutter ridge across normalized Doppler frequency and spatial frequency is defined as

$$\beta = \frac{2v_a T_p}{d},\tag{2.36}$$

where  $v_a$  is the velocity of the radar platform, d is the antenna separation, and  $T_p$  is the pulse repetition interval.

#### 2.2.5 STAP Filtering and Detection Theory Application

As described in [10],

$$Y = Y_I$$
,  $H_0$ : No Target Present  
 $Y = Y_I + \sigma_t a_{s,t}$ ,  $H_1$ : Target Present,

where  $Y_I$  is the response due to any interference,  $\alpha_t$  is the unknown target amplitude response, and  $a_{s,t}$  is the known unit amplitude system target response as derived previously.

Since the system interference is present regardless of target presence and the target presence is a shift of the data mean [10], the data will always have the covariance matrix  $R_I$  as defined previously.

Therefore, STAP processing is essentially matched filtering. The clutter and jamming characteristics are unknown in a scene. Therefore, filtering must be adaptive to cancel interference. The output of the matched filter bank is

$$z = w^H y, \tag{2.37}$$

where w is the filter weight vector, which is adaptive, and y is the space-time data snapshot. For fully adaptive STAP, weight vectors are optimized to filter data from specific Doppler frequency and angle cells individually. However, modifications of the STAP algorithm can be implemented to provide a reduced order representation of the weight vector. Modifications of the STAP algorithm can be found in [10]. For a non-adaptive, beam-formed response,

$$w = a_{s,t}.\tag{2.38}$$

Since the general form of the space-time steering vector is a basis vector for the discrete Fourier transform, the non-adaptive response can be computed using a twodimensional fast Fourier transform (FFT) [5].

There are several versions of STAP training to determine adaptive weight vectors. Common methods of adaptively computing the weight vectors include sample matrix inversion (SMI), subspace projection, or subspace SMI [10]. However, the choice of training approaches is dependent upon the system and will not be the focus of this chapter.

As derived in [5] the optimum weight vector for the simplest SMI approach is

$$w = \widehat{R}_I^{-1} a_{s,t}, \tag{2.39}$$

where  $\widehat{R}_{I}$  is the data covariance estimate achieved by the chosen training and estimation method.

The weighted space-time outputs are tested against a threshold, as shown in Figure 2.7, to determine if a target is present. This is done for every range bin to detect range, angle, and radial velocity of targets.

#### 2.2.6 Minimum Detectable Velocity

Minimum detectable velocity (MDV) is a common performance metric of STAP processing. MDV is determined by the minimum detectable Doppler (MDD), which can be discerned from the interference, especially clutter. Since the stationary clutter



Figure 2.7: High-level illustration of STAP.

is centered at zero Doppler, the upper MDD and lower MDD are defined in [7] as

$$MDD_{+} = \min_{f_{d}} \{ f_{d} | L_{SIR} \ge L_{0}, f_{d} > 0 \}$$
$$MDD_{-} = \min_{f_{d}} \{ f_{d} | L_{SIR} \ge L_{0}, f_{d} < 0 \}$$

where  $L_{SIR}$  is the signal-to-interference ratio loss and  $L_0$  is the maximum acceptable loss. These values are calculated for specific systems parameters using the radar range equation.

$$MDD = \frac{MDD_{+} - MDD_{-}}{2}$$
$$MDV = \frac{\lambda}{2}(MDD)$$

### CHAPTER 3

## MULTIPLE-INPUT SINGLE-OUTPUT SYSTEM IMPLEMENTATION

The MISO system is an unconventional radar system design. This system has some of the advantages of the SISO and SIMO systems but requires some of the additional considerations of the MIMO system. By using a single receiver, the MISO system can limit the received data rate. By using multiple transmitters, this system gains a degree of freedom to implement spatial processing. Since this system design approach requires the use of orthogonal waveforms, several modifications to the standard processing discussed in Chapter 2 must be made. In this chapter, several waveform designs will be proposed, and the modifications to the backprojection and STAP algorithms will be derived.

#### 3.1 Waveform Design

The waveform design examples discussed in this section are illustrated using the following Gotcha parameters [3].

$$f_c = 9.6 \text{GHz}$$
  
 $BW = 640 \text{MHz}$ 



Figure 3.1: FDMA Scheme of LFM chirps.

$$\alpha_{total} = 2 * 10^{13} \text{Hz/sec.}$$

These waveforms are intuitive options to achieve orthogonality with LFM processing. Future waveform design options are discussed in the Section 5.1.

## 3.1.1 FDMA

The frequency division multiple access (FDMA) scheme described in this section is shown in Figure 3.1.
A linear FM waveform is

$$s(t) = \exp\{j2\pi(\frac{\alpha}{2}t + f_c)t\}, \quad -\frac{T}{2} < t < \frac{T}{2}$$
(3.1)

FDMA over the bandwidth can be achieved for M transmit antennas, and each transmitter is indexed by m,

$$m = -\frac{M-1}{2}, ..., -1, 0, 1, ..., \frac{M-1}{2}.$$
(3.2)

The center frequency of each transmit waveform is

$$f_m = m \frac{BW}{M}.$$
(3.3)

Therefore, the transmitted waveforms of the FDMA scheme are defined as

$$s_m(t) = s(t) \exp\{j2\pi f_m t\}, \quad -\frac{T}{2} < t < \frac{T}{2}$$
(3.4)

$$= \exp\{j2\pi(\frac{\alpha}{2}t + f_c + f_m)t\}, \quad -\frac{T}{2} < t < \frac{T}{2}, \quad (3.5)$$

where,

$$\alpha = \frac{\alpha_{total}}{M} \tag{3.6}$$

$$T = \frac{BW/M}{\alpha} \tag{3.7}$$

The total bandwidth is

$$(f_c - \frac{BW}{2}) < f < (f_c + \frac{BW}{2}).$$
 (3.8)

The specific case of five transmitted signals for the Gotcha system is defined by:

$$m = -2, -1, 0, 1, 2$$
  
 $f_m = m \frac{640}{5} \text{MHz}$   
 $\alpha = \frac{2 * 10^{13}}{5} \text{Hz/sec}$ 



Figure 3.2: Frequency Wrapping Scheme of LFM chirps.

Therefore, (9600 - 320)MHz < f < (9600 + 320)MHz.

For example, the transmitter indexed by m = -2 is a linear FM chirp that starts at (9600 - 320) MHz and sweeps 640/5 = 128MHz. The chirp rate is  $\alpha = \frac{2*10^{13}}{5}$ Hz/sec; therefore, the pulse duration is

$$T = \frac{640/5 * 10^6}{2/5 * 10^{13}} = 32\mu \text{sec}$$

# 3.1.2 Frequency Wrapping

The frequency wrapping scheme described in this section is shown in Figure 3.2. This scheme follows a similar derivation as the FDMA scheme. However, each transmit waveform wraps the entire bandwidth for this scheme. The desired waveforms are of the form

$$s_m(t) = \exp\{j\theta_{\text{mod}}(t)\}, \quad -\frac{T}{2} < t < \frac{T}{2}.$$
 (3.9)

where  $\theta_{\text{mod}}$  has a starting frequency offset and wraps the full bandwidth for the pulse duration. This means  $\alpha = \alpha_{total}$  since we are not dividing the bandwidth between the transmitters.

For FDMA, we defined each waveform as

$$s_m(t) = \exp\{j2\pi(\frac{\alpha}{2}t + f_c + f_m)t\}, \quad -\frac{T}{2} < t < \frac{T}{2}$$
 (3.10)

$$= \exp\{j\theta(t)\}, \quad -\frac{T}{2} < t < \frac{T}{2}.$$
 (3.11)

where,

$$\theta(t) = 2\pi \left(\frac{\alpha}{2}t + f_m + f_c\right)t \tag{3.12}$$

$$f(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = \alpha t + f_m + f_c.$$
(3.13)

Therefore,

$$\theta(t) = 2\pi (f(t) - \frac{\alpha}{2}t)t.$$
(3.14)

To wrap the chirps within the bandwidth using the modulo function we define:

$$f_{\text{mod}}(t) = \text{mod}[(\alpha t + f_m + \frac{BW}{2}), BW] - \frac{BW}{2} + f_c$$
 (3.15)

 $f_{\text{mod}}(t)$  should range from  $(f_c - \frac{BW}{2})$  to  $(f_c + \frac{BW}{2})$ .



Figure 3.3: Frequency Wrapping Scheme of LFM chirps with varied chirp sweep rates.

 $\frac{BW}{2}$  is added and subtracted since the result of mod() is between 0 and BW.

$$\theta_{\text{mod}}(t) = 2\pi (f_{\text{mod}}(t) - \frac{\alpha}{2}t)t$$
(3.16)

$$= 2\pi (\mathrm{mod}[(\alpha t + f_m + \frac{BW}{2}), BW] - \frac{BW}{2} + f_c - \frac{\alpha}{2}t)t \qquad (3.17)$$

### 3.1.3 Varied Chirp Sweep Rates with Frequency Wrapping

The frequency wrapping scheme with varied chirp sweep rates described in this section is shown in Figure 3.3. This scheme follows the same derivation as the standard frequency wrapping scheme discussed above, but uses varied chirp sweep rates for the M transmit antennas. Therefore,

$$s_m(t) = \exp\{j\theta_{\text{mod}}(t)\}, \quad -\frac{T}{2} < t < \frac{T}{2},$$
(3.18)

where,

$$\theta_{\text{mod}}(t) = 2\pi (\text{mod}[(\alpha_m t + f_m + \frac{BW}{2}), BW] - \frac{BW}{2} + f_c - \frac{\alpha_m}{2}t)t$$
(3.19)

and

$$\alpha_m = m\alpha. \tag{3.20}$$

### 3.2 Orthogonality of the MISO Waveform Schemes

The proposed waveform schemes have both benefits and drawbacks to be considered. This section discusses the considerations of each waveform scheme to be used as orthogonal transmitting radar waveforms.

The auto-correlation and cross-talk of the waveform schemes are shown in the following graphs. There is no cross-talk in the FDMA scheme, as seen in Figure 3.4, while the cross-talk of the frequency wrapping scheme appears significant at first glance. This cross-talk, shown in Figure 3.5, is due to the fact that each waveform is a time-shifted version of the another waveform. The autocorrelation of the varied chirp sweep rate scheme results in multiple peaks, as shown in Figure 3.7, due to waveforms repeating across the pulse duration. Since we are only interested in the cross-talk of the waveforms over the stretch bandwidth, these undesired autocorrelation peaks and



Figure 3.4: Auto-Correlation and Cross-Talk of the FDMA Scheme of LFM chirps.

cross-correlation peaks will be ignored when calculating the range compressed phase history. However, this reduces the possible range extent of the scene. The autocorrelation and cross-correlation peaks are overlaid to show the possible range swath limitation in Figures 3.6 and 3.8. As shown, the frequency wrapped and varied sweep rate waveform schemes limit the usable range swath by a factor of  $\frac{1}{M}$ .

By observing the correlation of the three waveform schemes discussed here, some of the limitations can be seen. The frequency wrapping scheme and the varied sweep rate



Figure 3.5: Auto-Correlation and Cross-Talk of the Frequency Wrapping Scheme of LFM chirps.



Figure 3.6: Relevant Auto-Correlation and Cross-Talk of the Frequency Wrapping Scheme of LFM chirps.



Figure 3.7: Auto-Correlation and Cross-Talk of the Frequency Wrapping Scheme of LFM chirps with varied chirp sweep rates.

![](_page_45_Figure_0.jpeg)

Figure 3.8: Relevant Auto-Correlation and Cross-Talk of the Frequency Wrapping Scheme of LFM chirps with varied chirp sweep rates.

![](_page_46_Figure_0.jpeg)

Figure 3.9: Example MISO SAR System (for M=3 transmitting antennas).

scheme are both limited by the relationship between the range swath and number of transmit antennas. The FDMA scheme has ideal autocorrelation and cross-correlation characteristics; however, each waveform only spans a bandwidth of  $\frac{BW}{M}$  which limits the resolution of the individual channels by  $\frac{1}{M}$ .

### 3.3 MISO System SAR

MISO SAR uses a single receiver and multiple transmitters, as shown in Figure 3.9.

After choosing a waveform scheme, the discretized transmitted signals are described as

$$s_m[n] = s_m(nT_s) \tag{3.21}$$

where  $T_s$  is the sampling period and m denotes the transmitting antenna. The number of samples transmitted is  $N = floor(\frac{T}{T_s})$  and samples are indexed by n,

$$n = -\frac{N}{2}, \dots, \frac{N}{2}.$$

Just as described in the previous chapter, the number of samples on receive is given by  $L = \frac{t_{stop} - t_{start}}{T_s} = \frac{(t_{far} + T/2) - (t_{near} - T/2)}{T_s}$  and samples are indexed by l,

$$l = -\frac{L}{2}, \dots, \frac{L}{2}$$

A change of variables to let l be zero at scene center is applied for  $t = nT_s - T/2 = lT_s + t_{start}$ .

$$r_{g,m}[l] = r_{g,m}(lT_s) = \sigma_{g,m}s_m(lT_s - \Delta t_{g,m})$$
(3.22)

This is the received signal from the  $g^{th}$  scatterer from the  $m^{th}$  transmit waveform. The phase history over one slow time pulse, l, is

$$r[l] = \sum_{m=-\frac{M-1}{2}}^{\frac{M-1}{2}} \sum_{g=0}^{G} r_{g,m}[l].$$
(3.23)

The total phase history over all slow time pulses will be saved as a matrix of column vectors as shown below:

$$PH = [r_1[l], r_2[l], ..., r_N[l]]$$
(3.24)

where l=1, 2,...,L represent the slow time indices. The received signal is a coherent sum of the received signals of the  $g^{th}$  point scatterer due to each transmit antenna.

As described in Chapter 2, stretch processing can be performed for the MISO system by using multiple reference signals to separate the received signal into the orthogonal components of the M transmitters. This method was chosen for practical implementation due to the decrease in bandwidth resulting in a decreased sampling frequency requirement.

For each transmit antenna, a separate de-ramp reference signal is compared to the received signals. This reference signal is denoted as

$$w_m[l] = s_m^*(lT_s + t_{start} - \Delta t_{center,m}), \qquad (3.25)$$

where  $\Delta t_{center,m} = 2R_{center,m}/c$ . Note that this reference signal is over the entire receive period for one slow time pulse and there will be M of them as opposed to the single reference signal of the SISO or SIMO systems. For a received pulse from the  $m^{th}$  transmitter and  $g^{th}$  scatterer,  $r_{g,m}[l] = \sigma_g s_m (lT_s + t_{start} - \Delta t_{g,m})$ , the de-ramped receive signals are

$$d_{g,m}[l] = r_{g,m}[l]w_m^*[l]. ag{3.26}$$

Using stretch processing, the phase history received from orthogonal waveforms can be separated into the response of each transmit waveform. Therefore, the separate responses can be processed as multiple spatial channels. Since the two-way travel time for a point scatterer in the scene is identical for a MISO system and a SIMO system, the separate response of each transmitter can now be considered a virtual receive channel due to the reciprocity of the geometry.

#### 3.3.1 MISO System Backprojection

Back projected images can be produced for each spatial channel. The frequency wrapped and varied chirp sweep rate waveform schemes use the full bandwidth for each waveform. Therefore, an image of the same range resolution of a SISO system with the same bandwidth can be achieved with a single channel. The system model for the frequency wrapped and varied sweep rate waveform schemes is shown in Figure 3.10 For the FDMA scheme, the bandwidth of each channel is only  $\frac{BW}{M}$ , where M is the number of transmitting antennas. Therefore, the range resolution of each channel is degraded. In order to produce a full resolution image, each channel image is coherently added. FDMA MISO can be used to form an image with the same range

![](_page_49_Figure_0.jpeg)

Equivalent Full Resolution Images

Figure 3.10: MISO SAR Backprojection for frequency wrapped or varied sweep rates waveforms (for M=3 transmitting antennas).

resolution as a SISO system with the same bandwidth. The process of achieving full resolution images with FDMA waveforms is illustrated in Figure 3.11.

Only one of the channels is monostatic, while the remaining M-1 virtual channels are actually bistatic. For some radar systems, bistatic backprojection may need to be implemented. However, in the simulations described in this thesis, the monostatic range line approximation is used because the antenna separation is orders of magnitude smaller than the range to scene center. The geometry of the SAR geometry used justifies the use of monostatic backprojection for image formation. The bistatic range lines for the M transmitters and single receiver are shown from a bird's eye view of the system geometry in Figure 3.12. The simulation results discussed in this thesis assume a 45<sup>o</sup> elevation angle and 8000 meter elevation above the ground. The scene has a 100 meter radius about scene center. Therefore,  $R_3$  in Figure 3.12, the

![](_page_50_Figure_0.jpeg)

Figure 3.11: MISO SAR Backprojection for FDMA waveforms (for M=3 transmitting antennas).

slant range between the receiver and far range, is

$$R_3 = \sqrt{(8000)^2 + (8100)^2} \approx 11.385 \text{km.}$$
(3.27)

The antenna spacing of the simulation results is

$$d = \frac{\lambda}{2} = \frac{c}{2f_c} \approx 0.0156 \mathrm{m}, \qquad (3.28)$$

where  $f_c = 9.6$  GHz. For bistatic backprojection, the two-way path length would need to be taken into account. For example, the two-way path to far scene from the fifth transmitter in Figure 3.12 is  $R_5 + R_3$ . Monostatic backprojection can be applied for estimated range lines as shown in Figure 3.13. As shown in Table 3.1, the difference in bistatic two-way path lengths and the monostatic two-way path lengths are very small fractions of a wavelength. The time delay error associated with the two-way

![](_page_51_Figure_0.jpeg)

Figure 3.12: Bistatic Two-Way Paths (for M=5 transmitting antennas).

path length error,  $\Delta R$  is

$$\Delta \tau = \frac{2\Delta R}{c},\tag{3.29}$$

which is on the order of one hundred attoseconds  $(10^{-18})$ . By producing images with the monostatic antenna location, bistatic backprojection considerations can be ignored.

![](_page_52_Figure_0.jpeg)

Figure 3.13: Monostatic Two-Way Paths (for M=5 transmitting antennas).

Table	2.1	: M	Ionostatic	versus	Bistatic	Two-	Way	Path	Lengths	of	the	Simulated	Geometry.
-------	-----	-----	------------	--------	----------	------	-----	------	---------	----	-----	-----------	-----------

	Bistatic	Monostatic	Two-Way Path	$\Delta R$ as a
	Two-Way	Two-Way	Length Error	Fraction of
	Path Length	Path Length	(nm), $\Delta R$	Wavelength, $\frac{\Delta R}{\lambda}$
Transmitter 1	$R_1 + R_3$	$2\underline{R}_1$	21.442	$6.8615 * 10^{-7}$
Transmitter 2	$R_2 + R_3$	$2\underline{R}_2$	5.3587	$1.7148 * 10^{-7}$
Transmitter 3	$2R_3$	$2\underline{R}_3 = 2R_3$	0	0
Transmitter 4	$R_4 + R_3$	$2\underline{R}_4$	5.3587	$1.7148 * 10^{-7}$
Transmitter 5	$R_{5} + R_{3}$	$2\underline{R}_5$	21.442	$6.8615 * 10^{-7}$

![](_page_53_Figure_0.jpeg)

Figure 3.14: Receiving Array Wavefronts (for M=5 receiving antennas).

## 3.4 MISO System STAP

The STAP steering vectors developed in [7] and [10] hold for the frequency wrapped scheme and the varied chirp sweep rate scheme because the center frequency of each waveform is the same and the spatial geometry is the reciprocal of the typical SIMO system as shown in Figures 3.14 and 3.15. However, the FDMA scheme significantly increases the computational complexity of the STAP implementation.

Top-level MISO STAP is illustrated in Figure 3.16. In this thesis, the adaptive training method will not be discussed. The weight vector will be the space-time steering vector.

As will be derived in this section, spatial and temporal beamforming are no longer separable for FDMA waveforms. Therefore, the FDMA space-time steering vector will be derived separately from the steering vectors used to implement the space-time

![](_page_54_Figure_0.jpeg)

Figure 3.15: Transmitting Array Wavefronts (for M=3 transmitting antennas).

steering vectors for the frequency wrapped waveforms and the varied chirp sweep rate waveforms.

## 3.4.1 MISO STAP System Model

Since STAP focuses on moving targets, the Doppler frequencies for FDMA waveforms should be considered first. The Doppler of the target due to the wavelength of the  $m^{th}$  transmitter is

$$f_{d,m} = \frac{2v}{\lambda_m},\tag{3.30}$$

where  $\lambda_m = \frac{c}{f_c + f_m}$  and v is the radial velocity of the target. Following the traditional STAP derivation in [10], with modified variables to avoid confusion with previous equations, the received signal model from a single target with complex RCS  $\sigma$  for the FDMA scheme is

$$\tilde{r}_m(t) = \sigma u(t - \tau_m) \exp\{j 2\pi (f_c + f_m + f_{d,m})(t - \tau_m)\} \exp\{j \psi'\}, \quad (3.31)$$

![](_page_55_Figure_0.jpeg)

Figure 3.16: MISO STAP Diagram (for M=3 transmitting antennas).

where u is the complex envelope, the transmitting antenna is indexed by  $m = 0, ..., M - 1, \tau_m$  is the two-way time delay to the target from the  $m^{th}$  transmitter,  $f_c$  is the center frequency of the full transmitting bandwidth,  $f_m$  is the center frequency offset of the  $m^{th}$  transmitter, and  $\psi'$  is the arbitrary initial phase of the antennas. Note that the transmitting antennas are assumed to have synchronized initial phases. However, Section 3.5 will include the additional consideration of different initial phase terms,  $\psi'_m$ , caused by unsynchronized transmitters in the MISO radar system. This signal model differs from the SAR signal models for two reasons; the target is assumed to be moving and the waveforms are assumed to be narrowband. The narrowband assumption is part of the standard STAP model. Let  $\tau_m = \tau_0 - \tau'_m$ , where  $\tau_0$  is the two-way time delay for the reference antenna, m = 0, and  $\tau'_m$  is the difference in the two-way time delay between the reference transmitter and the  $m^{th}$  transmitter. Assuming the target is at a range  $R_t$  from the reference transmitter

$$\tau_0 = \frac{2R_t}{c} \tag{3.32}$$

The relative delay for the  $m^{th}$  transmitter is

$$\tau'_m = -m\frac{d}{c}\cos\theta\sin\phi,\tag{3.33}$$

where d,  $\theta$ , and  $\phi$  are the antenna separation, elevation angle, and azimuth angle, respectively. By substituting this two-way time delay, the signal model becomes

$$\tilde{r}_m(t) = \sigma u(t - \tau_m) \exp\{j2\pi (f_c + f_m + f_{d,m})(t - \tau_0 - \tau'_m)\} \exp\{j\psi'\}$$
(3.34)

Define a spatial frequency,  $f_{s,m}$ , of the  $m^{th}$  transmitter from the spatial phase term such that

$$\exp\{jm2\pi f_{s,m}\} = \exp\{-j2\pi (f_c + f_m)\tau'_m\}.$$
(3.35)

Resulting in a spatial frequency,

$$f_{s,m} = -(f_c + f_m) \frac{\tau'_m}{m}$$
 (3.36)

$$= (f_c + f_m)\frac{d}{c}\cos\theta\sin\phi \qquad (3.37)$$

$$= d\cos\theta\sin\phi/\lambda_m. \tag{3.38}$$

Therefore, the FDMA waveform scheme introduces an additional transmitter dependence on the traditional spatial frequency and Doppler frequency that was not present in the traditional SIMO STAP described in Chapter 2.

As described in [10], assume  $u(t - \tau_m) \approx u(t - \tau_0)$ . This approximation is due to the narrowband waveform assumption which implies that the relative time delay is insignificant within the complex envelope. Additionally,  $\tau'_m$  is assumed to be very small (possibly on the order of picoseconds) and  $f_{d,m}$  is typically relatively small (possibly tens of Hz). Therefore,

$$f_{d,m}\tau'_m \approx 0. \tag{3.39}$$

In order to separate the terms which vary in slow time or in the spatial domain from the non-varying terms, let

$$\exp\{j\psi\} = \exp\{j\psi'\} \exp\{j2\pi(-f_c\tau_0 - f_{d,m}\tau_0)\}$$
(3.40)

The system model becomes

$$\begin{split} \tilde{r}_{m}(t) &= \sigma \exp\{j\psi\}u(t-\tau_{0})\exp\{jm2\pi f_{s,m}\}\exp\{j2\pi f_{d,m}t\}\exp\{j2\pi f_{c}t\}...\\ &\exp\{j2\pi(f_{m}t-f_{m}\tau_{0})\}\\ &= \sigma \exp\{j\psi\}u(t-\tau_{0})\exp\{jm2\pi f_{s,m}\}\exp\{j2\pi f_{d,m}t\}\exp\{j2\pi(f_{c}+f_{m})t\}...\\ &\exp\{-j2\pi f_{m}\tau_{0}\}. \end{split}$$

Down-conversion produces

$$r_m(t) = \tilde{r}_m(t) \exp\{-j2\pi (f_c + f_m)t\}$$
  
=  $\sigma \exp\{j\psi\} \exp\{jm2\pi f_{s,m}\} \exp\{-j2\pi f_m\tau_0\} u(t-\tau_0) \exp\{j2\pi f_{d,m}t\}.$ 

After matched filtering, the model is

$$x_{m}(t) = \sigma \exp\{j\psi\} \exp\{jm2\pi f_{s,m}\} \exp\{-j2\pi f_{m}\tau_{0}\}...$$
$$\sum_{n=0}^{N-1} \exp\{j2\pi n f_{d,m}T_{p}\} X(t-\tau_{0}-nT_{p}, f_{d,m}T_{p}),$$

where  $T_p$  denotes the pulse repetition interval (PRI) and  $X(\tau, f)$  is the waveform ambiguity function defined in [10] as

$$X(\tau, f) = \int_{-\infty}^{\infty} u_p(\beta) u_p^*(\beta - \tau) \exp\{j2\pi f\beta\} d\beta, \qquad (3.41)$$

where X(0,0) = 1. Let  $t_n = \tau_0 + nT_p$ , where n = 0, ..., N - 1. Then, let  $x_{mn}$  denote  $x_m(t_n)$ . The system model becomes

$$x_{mn} = \sigma \exp\{j\psi\} \exp\{jm2\pi f_{s,m}\} \exp\{-j2\pi f_m \tau_0\}...$$
$$X(t - \tau_0 - nT_p, f_{d,m}T_p) \exp\{j2\pi nf_{d,m}T_p\}.$$

The next step, following the derivation of [10], is to assume that the time-bandwidth product of the pulse waveform and the expected range of the target Doppler frequency are such that the waveform is insensitive to the target Doppler shift. That means  $X(0, f) \approx 1$ . Let  $A = \sigma \exp\{j\psi\}$  denote the complex random amplitude of the model. Then,

$$x_{mn} = A \exp\{jm2\pi f_{s,m}\} \exp\{jn2\pi f_{d,m}T_p\} \exp\{-j2\pi f_m\tau_0\}$$
(3.42)

$$= A \exp\{jm2\pi f_{s,m}\} \exp\{jn2\pi (2v(f_c + f_m)/c)T_p\} \exp\{-j2\pi f_m\tau_0\}.(3.43)$$

	spatial	Doppler	range-dependent
	frequency	frequency	phase ramp
traditional	$f_s = d\cos\theta\sin\phi/\lambda$	$f_d = \frac{2v}{\lambda}$	N/A
FDMA	$f_{s,m} = d\cos\theta\sin\phi/\lambda_m$	$f_{d,m} = \frac{2v}{\lambda_m}$	$\exp\{-j2\pi f_m\tau_0\}$

Table 3.2: STAP Implications of the Proposed Waveform Schemes.

Following a similar derivation, the typical STAP model for a MISO system with a frequency wrapped waveform scheme or a varied chirp sweep rate scheme is

$$x_{mn} = A \exp\{jm2\pi f_s\} \exp\{jn2\pi f_d T_p\},$$
(3.44)

where  $f_d = \frac{2v}{\lambda}$ ,  $f_s = d\cos\theta\sin\phi/\lambda$ , and  $\lambda = \frac{c}{f_c}$ .

By comparing the two models, the implications of the FDMA waveform scheme on the STAP model are listed in Table 3.2.

The spatial dependence, due to the different center frequencies of the M orthogonal FDMA waveforms, of the Doppler term results in the inseparability of the spatial and temporal components as opposed to typical STAP. The implications of the spatial and Doppler frequency terms for FDMA waveforms will be discussed in the following subsections.

The range dependent phase ramp due to the  $\exp\{-j2\pi f_m \tau_0\}$  term results in a unique consideration not present in the traditional SIMO STAP. In order to examine this term, let

$$f_m = m f_1, \tag{3.45}$$

where  $f_1$  is the reference offset frequency between two adjacent transmitting antennas. Note that for traditional STAP this phase ramp term is eliminated because  $f_1$  is zero. This phase ramp term rotates the range constant angle of arrival (AOA) phase ramp

![](_page_60_Figure_0.jpeg)

Figure 3.17: Additive Range Dependent Phase Ramp for FDMA STAP.

of the M spatial channels across range slices. The period of this rotation can be seen by letting

$$\tau_0 = \frac{K}{f_1},\tag{3.46}$$

where K is any integer. Inserting Equation 3.45 and Equation 3.46 into the phase ramp term, produces

$$\exp\{-j2\pi mK\}\tag{3.47}$$

Therefore, the phase ramp completes a full rotation of  $2\pi$  with a period of  $\frac{1}{f_1}$ . Let  $\Theta = mod(f_1\tau_0, 2\pi)$ . Then,  $\exp\{-j2\pi m f_1\tau_0\} = \exp\{-j2\pi m\Theta\}$ . As  $R_t$  increases,  $\tau_0$  increases, which also increases  $\Theta$ . This phase ramp depends on m as shown in Figure 3.17. For the simulated parameters discussed in this thesis, where M = 5,  $f_1 = \frac{640}{5}$ MHz = 128MHz. Therefore, the range dependent phase ramp has a period

of  $\frac{1}{f_1} \approx 7.8$ nsec. In terms of range, since  $R_t = \frac{c\tau_0}{2}$ , the phase ramp completes a full  $2\pi$  rotation approximately every 1.17 meters.

## 3.4.2 MISO System Spatial Steering Vector

As previously discussed, the concept of separable spatial and temporal steering vectors does not apply to FDMA. However, the frequency wrapped waveforms and varied chirp sweep rate waveforms produce the same spatial and temporal steering vectors as discussed in the Chapter 2 due to the narrowband assumption of STAP. The spatial steering vector is denoted as

$$\mathbf{a}_{\mathbf{s}}(f_s) = [1, e^{j2\pi f_s}, \dots, e^{j2\pi(M-1)f_s}]^T$$
(3.48)

where  $f_s = d \cos \theta \sin \phi / \lambda$  is the spatial frequency and d,  $\theta$ , and  $\phi$  are the antenna separation, elevation angle, and azimuth angle, respectively. The spatial steering vector is the  $\exp\{jm2\pi f_s\}$  term in the STAP model of Equation 3.44.

For the frequency wrapped waveform and varied chirp sweep rate waveform schemes, the center frequency of each antenna is the same, and the phase delay associated with the wavefront is the same due to geometry reciprocity. Therefore, the frequency wrapped waveform and varied chirp sweep rate schemes can be spatially beamformed using the traditional spatial steering vector which is a basis vector of the discrete Fourier transform (DFT).

For the FDMA scheme, the spatial steering needs to account for the different center frequencies of each waveform. The wavelength variation cannot be factored out independent of the angle of arrival, defined by  $\theta$  and  $\phi$ . Therefore, the spatial steering of FDMA is no longer a basis vector of the DFT and will require the use of a matched filter bank as well as correcting for the phase ramp described by the

![](_page_62_Figure_0.jpeg)

Figure 3.18: Range Dependence of FDMA Spatial Steering.

 $\exp\{-j2\pi f_m \tau_0\}$  term of Equation 3.42. The range dependence of FDMA STAP is shown in Figures 3.18 and 3.19. For comparison, the typical STAP steering vector, which is applied to the frequency wrapped and varied sweep rate MISO systems, is shown as across range in Figures 3.20 and 3.21.

![](_page_63_Figure_0.jpeg)

Figure 3.19: Range Dependence of FDMA Spatial Steering (Top View).

![](_page_64_Figure_0.jpeg)

Figure 3.20: Typical Spatial Steering Across Range.

![](_page_65_Figure_0.jpeg)

Figure 3.21: Typical Spatial Steering Across Range (Top View).

### 3.4.3 MISO System Temporal Steering Vector

As derived in the background chapter, the temporal steering vector is typically denoted as

$$\mathbf{a}_{\mathbf{t}}(\tilde{f}_d) = [1, e^{j2\pi \tilde{f}_d}, ..., e^{j2\pi(N-1)\tilde{f}_d}]^T$$
(3.49)

where  $\tilde{f}_d = \frac{2vT_p}{\lambda} = \frac{f_d}{f_p}$  is the normalized Doppler frequency.

The frequency wrapped waveform and varied chirp sweep rate waveform schemes can be temporally beamformed using the traditional temporal steering vector, which is a basis vector of the DFT. The temporal steering vector is the  $\exp\{jn2\pi f_d T_p\}$  term in the STAP model of Equation 3.44 due to the narrowband assumption of STAP.

For the FDMA scheme, the a temporal steering vector would need to be altered such that

$$\mathbf{a}_{\mathbf{t}}(f_d) = [1, e^{j2\pi f_{d,m}T_p}, \dots, e^{j2\pi(N-1)f_{d,m}T_p}],$$
(3.50)

where  $f_{d,m} = \frac{2v}{\lambda_m}$ . However, the center frequency dependence of the Doppler frequency would require a separate temporal steering vector for each spatial element (m). Therefore, the temporal steering vector has spatial dependence and is no longer a basis vector of the DFT. As mentioned previously, this negates the concept of separable steering vectors and separable spatial and temporal beamforming. Therefore, the FDMA space-time beamforming will be performed using Equation 3.42 rather than applying the concept of separable spatial and temporal steering.

### 3.4.4 MISO System Space-Time Steering Vector

The space-time steering vector for the frequency wrapped waveform and varied chirp sweep rate waveform schemes is the typical derivation discussed in the background chapter. This typical space-time steering vector is defined as

$$\mathbf{a}_{\mathbf{s},\mathbf{t}} = \mathbf{a}_{\mathbf{t}} \otimes \mathbf{a}_{\mathbf{s}},\tag{3.51}$$

where  $\otimes$  represents the Kronecker matrix product. Therefore, STAP processing can be implemented using a two-dimensional DFT for the frequency wrapped waveform and varied chirp sweep rate waveform schemes.

The center frequency dependence and additive range dependent phase ramp alters the derivation for the FDMA waveforms. The traditional STAP steering vectors assume separability of the spatial and temporal domains. Due to the additional terms included in Equation 3.42, FDMA STAP can be applied using a matched filter bank. The space-time steering vector is the model of the expected response of a target at a certain angle of arrival (AOA) and Doppler frequency.

Since the wavelength associated with the center frequency of each spatial channel is different, the space-time steering vector is defined to depend on radial target velocity rather than Doppler frequency to avoid confusion.

The FDMA space-time steering vector is

$$\mathbf{a_{s,t}}(f_{s,m},v) = [1,...,e^{jm2\pi f_{s,m}}e^{j2\pi f_m\tau_0}e^{jn2\pi T_p 2v(f_c+f_m)/c},...,$$
(3.52)

$$e^{j(M-1)2\pi f_{s,m}} e^{j2\pi f_m \tau_0} e^{j(N-1)2\pi T_p 2v(f_c + f_m)/c}]^T.$$
(3.53)

For space-time beamforming, the space-time steering vector is applied as matched filter to each range slice separately. The output of the matched filter bank is

$$z = w^H y, \tag{3.54}$$

where  $w = \mathbf{a}_{s,t}$  is the filter weight vector, y is the received space-time data snapshot vector for the chosen range slice, and  $\{\}^H$  denotes the Hermitian transpose.

As a caution, the clutter covariance for FDMA will not follow the derivation discussed in the background as described in [10]. The FDMA waveform scheme will increase the clutter covariance rank by a factor of M as described in [1] and [6].

## 3.5 MISO System Simulation of the Gotcha Radar System

FDMA MISO SAR can be demonstrated using the 2006 Gotcha SAR Based GMTI Challenge Problem data set supplied by the Sensors ATR Division of the Air Force Research Laboratory [9]. This data set was collected using three receive phase centers and one transmitter. In order to provide a manageable dataset, the range gating described in Figure 3.22 was applied to produce 384 range bins across the 71 second data set. The moving target of interest in the Gotcha data set is a Dodge Durango. Since the Durango moves across range bins, the 384 range bins were chosen differently for each slow time pulse to center around the Durango location. Range gating as a function of time is shown in Figure 3.23. The motion of the Dodge Durango is graphed in Figure 3.24.

The notional MISO radar system design using FDMA waveforms is shown in Figure 3.25. The three transmit antennas are separated by a distance d, and the response of each waveform is received at one receive antenna. The phase history is range compressed using stretch processing with the three original transmit waveforms as reference signals. This will also separate the contribution of each transmit waveform into three virtual receive channels. At this point in the process, the three separate range compressed phase histories are equivalent to a SIMO system of the

![](_page_69_Figure_0.jpeg)

Figure 3.22: Gotcha Range Gating [9].

![](_page_69_Figure_2.jpeg)

Figure 3.23: Gotcha Range Gating as a Function of Time [9].

![](_page_70_Figure_0.jpeg)

Figure 3.24: Speed and Position Relative to Start Position of Durango in Gotcha Data Set [9].

same antenna spacing. The SAR imaging is then implemented by back-projecting the three virtual receive channels over a chosen slow time dwell and coherently summing them for the full resolution image of a SISO system of the same parameters. The STAP is applied to the CPI of the M = 3 virtual receive channels for the chosen number of slow time samples, N.

In order to apply these concepts to the real world Gotcha data set, the following method, shown in Figure 3.26, was implemented to simulate the MISO radar system using this SIMO system data. The three receiving antennas are separated by a distance *d*. The bandwidth of the three receive channels are separated into the three bandwidths which would have been separately transmitted in the MISO system waveforms. The first bandwidth (approximately 9.28 GHz to 9.49 GHz) is used from the first receive channel, the second bandwidth (approximately 9.49 GHz to 9.7 GHz) is used from the second receive channel, and the third bandwidth (approximately 9.7

![](_page_71_Figure_0.jpeg)

Figure 3.25: Notional MISO Radar System Design.
GHz to 9.92 GHz) is used from the third receive channel. After separating the bandwidths of the three receive signals, range compression is performed. At this point in the process, the three receive channel outputs are equivalent to the three virtual receive channel outputs of the FDMA MISO system. Therefore, the SAR image processing and the STAP are implemented using the previously mentioned approaches.

However, this method of separating the frequency band of the received signal will cause phase incoherence, which must be corrected in order to perform space-time beamforming. The transmitted pulse of the SIMO Gotcha radar, with initial phase  $\psi'$  is

$$s(t) = \exp\{j2\pi(\frac{\alpha}{2}t^2 + f_c t)\}\exp\{j\psi'\}u(t), \qquad (3.55)$$

where

$$u(t) = \sum_{n=1}^{N} u_p(t - T_p), \qquad (3.56)$$

$$u_p(t) = rect(\frac{t-\frac{T}{2}}{T}),$$
 (3.57)

and  $T_p$  and T are the PRI and the pulse width, respectively. By selecting separate bands of the received signal as described, the starting phase of each band cannot be assumed to be identical, as was done for the simulated MISO data. The method of selecting bands from each received pulse also applies a shift to the starting time of the waveform. Since radar observes linear scattering, the transmitted frequency is equal to the received frequency. Therefore, the effect of selecting frequency bands on receive can be illustrated by observing the related transmitted waveforms. Assuming the original transmitted pulse was separated into M FDMA waveforms, as shown in Figure 3.27, the transmitted pulse of the  $m^{th}$  transmitter is defined as

$$s_m(t) = s(t)rect(\frac{t - \frac{T}{2M}(2m+1)}{\frac{T}{M}})$$
 (3.58)



Figure 3.26: Real Data Simulation of the MISO Radar System Design.



Figure 3.27: Separation of Gotcha pulses for MISO FDMA Equivalency.



Figure 3.28: Rectangular Pulse Used to Define Bands of the Gotcha Chirp.

$$= s(t)rect(\frac{Mt}{T} - \frac{1}{2}(2m+1)), \qquad (3.59)$$

so that

$$s(t) = \sum_{m=0}^{M} s_m(t)$$
 (3.60)

As shown in Figure 3.28, the leading edge of the  $rect(\frac{Mt}{T} - \frac{1}{2}(2m+1))$  function is at

$$t = \frac{mT}{M}.\tag{3.61}$$

Let  $T_m$  denote the start time of the  $m^{th}$  transmitter. Then,

$$T_m = \frac{mT}{M}.$$
(3.62)

Since the pulse duration is related to the bandwidth and chirp sweep rate, such that  $T = \frac{BW}{\alpha}$ , the start time of the  $m^{th}$  transmitter is

$$T_m = \frac{mBW}{M\alpha}.$$
(3.63)

The starting phase of the  $m^{th}$  transmitter,  $\psi'_m$ , is

$$2\pi(\frac{\alpha}{2}T_m^2 + f_c T_m) + \psi' = \psi'_m.$$
(3.64)

Define a change in phase relative to the starting phase of the reference transmitter,  $\psi'$ , as

$$\psi_m = 2\pi (\frac{\alpha}{2}T_m^2 + f_c T_m), \qquad (3.65)$$

so that

$$\psi'_m = \psi' + \psi_m. \tag{3.66}$$

This time delay and phase delay must be accounted for when applying FDMA MISO STAP to the Gotcha data. For the Gotcha data set, the starting time delay of the  $m^{th}$  virtual transmitted pulse is

$$T_m = \frac{m(640 * 10^6)}{3(2.133 * 10^{13})} \approx (1.0002 * 10^{-5})m.$$
(3.67)

The phase delay and start time delay of each of the three bands are displayed in Table 3.3.

After computing the phase delay and time delay for the Gotcha implementation, the original FDMA STAP model described in Equation 3.68 will need to be changed

	Start Time Delay $T_m$ ( $\mu$ s)	Phase Delay $\psi_m$	
m = 0	0	0	
m = 1	10.002	$2\pi 9.7082 * 10^4 = 6.0998 * 10^5$	
m = 2	20.003	$2\pi 1.9630 * 10^5 = 1.2334 * 10^6$	

Table 3.3: Representative Gotcha FDMA Considerations for STAP.

for the Gotcha data set when modeling the FDMA MISO STAP. The received signal model would be

$$\tilde{r}_m(t) = \sigma u(t - \tau_m - T_m) \exp\{j2\pi (f_c + f_m + f_{d,m})(t - \tau_m - T_m)\} \exp\{j\psi'_m\}, \quad (3.68)$$

where u is the complex envelope, the transmitting antenna is indexed by  $m = 0, ..., M - 1, \tau_m$  is the two-way time delay to the target from the  $m^{th}$  transmitter,  $f_c$  is the center frequency of the full transmitting bandwidth,  $f_m$  is the center frequency offset of the  $m^{th}$  transmitter,  $T_m$  is the starting time delay associated with the  $m^{th}$  transmitter, and  $\psi'_m$  is the initial phase of the  $m^{th}$  transmitter. Note that the narrowband model is being considered, just as described in [10]. As derived in Section 3.4.1, the  $\tau_m = \tau_0 - \tau'_m$  substitution produces a signal model of

$$\tilde{r}_m(t) = \sigma u(t - \tau_m - T_m) \exp\{j2\pi (f_c + f_m + f_{d,m})(t - \tau_0 - \tau'_m - T_m)\} \exp\{j\psi'_m\}$$
(3.69)

The spatial frequency,  $f_{s,m}$ , of the  $m^{th}$  transmitter from the spatial phase term will still be defined such that

$$\exp\{jm2\pi f_{s,m}\} = \exp\{-j2\pi (f_c + f_m)\tau'_m\}.$$
(3.70)

Therefore, the spatial frequency is

$$f_{s,m} = -(f_c + f_m)\frac{\tau'_m}{m}$$

$$= (f_c + f_m)(\frac{d}{c}\cos\theta\sin\phi)$$
$$= d\cos\theta\sin\phi/\lambda_m$$

Assume  $u(t - \tau_m - T_m) \approx u(t - \tau_0 - T_m)$ . This assumption is due to the narrowband waveform model. The previous assumption,  $f_{d,m}\tau'_m \approx 0$ , remains. In order to separate the terms which vary in slow time or in the spatial domain from the non-varying terms, let

$$\exp\{j\psi\} = \exp\{j\psi'\} \exp\{j2\pi(-f_c\tau_0 - f_{d,m}\tau_0)\}$$
(3.71)

The system model becomes

$$\tilde{r}_{m}(t) = \sigma \exp\{j\psi\} \exp\{j\psi_{m}\} u(t - \tau_{0} - T_{m}) \exp\{jm2\pi f_{s,m}\}...$$
$$\exp\{j2\pi f_{d,m}t\} \exp\{j2\pi (f_{c} + f_{m})t\} \exp\{-j2\pi f_{m}\tau_{0}\}...$$
$$\exp\{-j2\pi (f_{c} + f_{m} + f_{d,m})T_{m}\}.$$

Down-conversion produces

$$r_{m}(t) = \tilde{r}_{m}(t) \exp\{-j2\pi(f_{c}+f_{m})t\} \exp\{-j\psi_{m}\}$$
  
=  $\sigma \exp\{j\psi\}u(t-\tau_{0}) \exp\{jm2\pi f_{s,m}\} \exp\{j2\pi f_{d,m}t\}...$   
$$\exp\{-j2\pi f_{m}\tau_{0}\} \exp\{-j2\pi(f_{c}+f_{m}+f_{d,m})T_{m}\}.$$

After matched filtering with the shifted complex envelope,  $u(t - \tau_0 - T_m)$ , the model is

$$x_{m}(t) = \sigma \exp\{j\psi\} \exp\{jm2\pi f_{s,m}\} \exp\{-j2\pi f_{m}\tau_{0}\}...$$
$$\sum_{n=0}^{N-1} \exp\{j2\pi n f_{d,m}T_{p}\} X(t-\tau_{0}-nT_{p}, f_{d,m}T_{p})$$

where  $T_p$  denotes the pulse repetition interval (PRI) and  $X(\tau, f)$  is the waveform ambiguity function. Just as described in Section 3.4.1 for the synchronized FDMA system, the slow time pulses are defined such that  $t_n = \tau_0 + nT_p$ . Then, let  $x_{mn}$  denote  $x_m(t_n)$ . The system model becomes

$$x_{mn} = \sigma \exp\{j\psi\} \exp\{jm2\pi f_{s,m}\} \exp\{-j2\pi f_m \tau_0\}...$$
$$X(t - \tau_0 - nT_p, f_{d,m}T_p) \exp\{j2\pi nf_{d,m}T_p\}.$$

Just as before, the time-bandwidth product of the pulse waveform and the expected range of the target Doppler frequency are assumed to be such that the waveform is insensitive to the target Doppler shift. That means  $X(0, f) \approx 1$ . Let  $A = \sigma \exp\{j\psi\}$ denote the complex random amplitude of the model. Then,

$$x_{mn} = A \exp\{jm2\pi f_{s,m}\} \exp\{jn2\pi f_{d,m}T_p\} \exp\{-j2\pi f_m\tau_0\}$$
$$= A \exp\{jm2\pi f_{s,m}\} \exp\{jn2\pi (2v(f_c + f_m)/c)T_p\} \exp\{-j2\pi f_m\tau_0\}.$$

Therefore, the FDMA STAP model derived in Section 3.4.1 will apply to the MISO FDMA Gotcha model without any additional phase offset terms.

### CHAPTER 4

# MULTIPLE-INPUT SINGLE-OUTPUT SYSTEM RESULTS

The concepts discussed in this thesis were simulated on ideal point scatterers and real world Gotcha data using the method described in the Chapter 3. The results produced by the simulation and using the Gotcha data are presented in this chapter.

### 4.1 MISO SAR

The FDMA simulation images are shown in Figures 4.1, 4.2, 4.3, 4.4, 4.5, and 4.6, where Figure 4.6 shows the coherent sum of the five channels. Range is along the x-axes and cross-range is along the y-axes. As expected, the coherent sum has an improved point spread in the range dimension due to increasing the total bandwidth. The frequency wrapped and varied sweep rate waveform schemes produce the same result without needing to coherently sum.

Using the Gotcha data set, the following images were produced. Figures 4.7, 4.8, and 4.9 illustrate the low-resolution images produced by the three FDMA bands. In order to produce these images, the phase histories collected at each receiver were separated into three bands. The first band of the first channel produced Figure 4.7. The second band of the second channel produced figure 4.8. The third band of the

third channel produced figure 4.9. Due to the reciprocity of the geometry of a MISO system and a SIMO system, this method simulates the same phase delay of the FDMA MISO system. The coherently added image is shown in Figure 4.10. For comparison, a full resolution image produced from channel 1 is shown in Figure 4.11

### 4.2 MISO STAP

The typical space-time beamforming, as described in [10], was implemented on simulated data using the frequency wrapped waveform scheme. An example stationary target at  $0^0$  azimuth is shown in Figure 4.12. The axes are indexed to display spatial frequency and Doppler frequency.

The matched filter approach for space-time beamforming with FDMA waveforms, as described in the implementation chapter, was implemented using a FDMA MISO simulation. The results for a stationary point scatterer at  $0^0$  azimuth are shown in Figure 4.13. As shown through these simulations, the FDMA STAP produces comparable results. However, it is no longer logical to plot results as a function of normalized Doppler frequency since the Doppler shift is slightly different depending on the transmitting frequency of each spatial channel. Therefore, the results are shown as a function of spatial frequency and radial target velocity.

The response of a moving point scatterer with a radial slant plane velocity of 10 m/s toward the radar is also shown in Figure 4.14. Note that, as described by the geometry in the background chapter, the radial velocity is the portion of the velocity in the slant plane.

The response of a moving point scatterer located at approximately 50 meters from scene center in the cross-range dimension with a radial slant plane velocity of 10 m/s toward the radar is also shown in Figure 4.15. Note that, as described by the geometry in the background chapter, the radial velocity is the portion of the velocity in the slant plane.

For comparison, the traditional space-time beamforming was applied to the SIMO Gotcha system data set to represent the expected detection. As shown in Figure 3.24, the Durango is moving at approximately 13 m/s around 40 seconds. Therefore, beamforming was applied to the  $40f_p = 86864$  starting pulse for 100 pulses. The Durango is located in the center range bin as shown in Figure 3.23. The result of traditional SIMO space-time beamforming is shown in Figure 4.16 as a function of spatial frequency and normalized Doppler frequency. The moving target is labeled in Figure 4.17 as a function of radial velocity and sine of azimuth angle relative to scene center. The slant plane radial velocity of the target is approximately 9 m/s, which corresponds to a ground plane target speed of approximately 12.8 m/s. The clutter ridge slope can be calculated using Equation 2.36 and results in a slope of 0.1859. The clutter ridge slope is shown in Figure 4.18.

The space-time beamforming derived for the FDMA MISO system was also applied to the Gotcha data set to demonstrate the feasibility of STAP for this radar system design. The results are shown in Figure 4.19. The CPI was formed as described in the previous chapter. As shown, the results are similar, and the Durango is detected at the same angle of arrival and radial target velocity.



Figure 4.1: Back Projected Image for Channel 1 of the FDMA Simulation.



Figure 4.2: Back Projected Image for Channel 3 of the FDMA Simulation.



Figure 4.3: Back Projected Image for Channel 5 of the FDMA Simulation.



Figure 4.4: Back Projected Image for Channel 2 of the FDMA Simulation.



Figure 4.5: Back Projected Image for Channel 4 of the FDMA Simulation.



Figure 4.6: Coherently Summed Back Projected Image of the FDMA Simulation.



Figure 4.7: Back Projected Image for Band 1 of the FDMA MISO Gotcha.



Figure 4.8: Back Projected Image for Band 2 of the FDMA MISO Gotcha.



Figure 4.9: Back Projected Image for Band 3 of the FDMA MISO Gotcha.



Figure 4.10: Coherently Summed Back Projected Image of the FDMA MISO Gotcha.



Figure 4.11: Example Full Resolution Back Projected Image of Gotcha Channel 1.



Figure 4.12: Example Space-Time Beamforming of the Frequency Wrapped Waveform Simulation (stationary point scatterer at  $0^0$  azimuth).



Space-Time Beamforming of FDMA Simulation (using matched filters)

Figure 4.13: Example Space-Time Beamforming of the FDMA Waveform Simulation (stationary point scatterer at  $0^0$  azimuth).



Space-Time Beamforming of FDMA Simulation (using matched filters)

Figure 4.14: Example Space-Time Beamforming of the FDMA Waveform Simulation (point scatterer at  $0^0$  azimuth moving 10 m/s towards the radar).



Space-Time Beamforming of FDMA Simulation (using matched filters)

Figure 4.15: Example Space-Time Beamforming of the FDMA Waveform Simulation (point scatterer at  $0.35^0$  azimuth moving 10 m/s towards the radar).



Figure 4.16: Traditional SIMO Space-Time Beamforming of Gotcha MTI Data Set.



Figure 4.17: Traditional SIMO Space-Time Beamforming of Gotcha MTI Data Set with the Detected Target Marked.



Figure 4.18: Traditional SIMO Space-Time Beamforming of Gotcha MTI Data Set with the Slope of the Clutter Ridge Marked.



Figure 4.19: Space-Time Beamforming of the Gotcha Data Simulation of the MISO Radar System Design.

# CHAPTER 5

### CONCLUSIONS

The MISO SAR system is a plausible system design for gaining spatial information without increasing the volume of received data. The waveform design is crucial to the success of this radar system, and the trade-offs must be carefully considered. The trade-offs discussed in this thesis are presented in Table 5.1.

The FDMA scheme has ideal autocorrelation and cross-correlation characteristics. Each FDMA waveform only spans a bandwidth of  $\frac{BW}{M}$  which limits the resolution by  $\frac{1}{M}$  for STAP. However, the full resolution can be achieved for backprojection imaging by coherently summing the M channels. The center frequencies are different for each waveform, which complicates the STAP steering vectors. The frequency wrapping scheme and the varied sweep rate scheme are both limited by the relationship between the range swath and number of transmit antennas. For the frequency wrapping scheme, the undesired cross-correlation peaks infringe on the orthogonality over the scene size at  $\frac{T_p}{M}$ . For the varied sweep rate scheme, the undesired autocorrelation peaks infringe on the orthogonality over the scene size at  $\frac{T_p}{M}$ . Therefore, the unambiguous range swath is reduced by a factor of  $\frac{1}{M}$ .

The traditional STAP derivation must be modified for FDMA waveforms and the notion of separable spatial and temporal steering vectors is no longer applicable.

	STAP	Swath	STAP	Clutter
	Resolution	Size	Implementation	Covariance Rank
FDMA	$\frac{1}{M}$	1	MF Bank	М
Wrapped	1	$\frac{1}{M}$	FFT $\checkmark$	1
Varied Sweeps	1	$\frac{1}{M}$	FFT $\checkmark$	1

Table 5.1: Considerations of the Proposed Waveform Schemes.

Therefore, FDMA requires matched filtering against each angle of arrival and Doppler frequency instead of using the two-dimensional FFT. However, if the accuracy of Doppler frequency is able to be compromised to save on computational complexity, the traditional space-time beamforming may still be preferable for FDMA with nearly negligible wavelength offset between the spatial channels but the additional range dependent phase ramp will need to be corrected for separately. However, using the traditional FFT computational method for FDMA waveforms is a design choice which will be affected by the percent bandwidth  $\left(\frac{BW}{f_c}\right)$  of the system. The higher the percent bandwidth of the radar system, the less effective the traditional FFT approach will be. For a system such as Gotcha, with a percent bandwidth of approximately 6.7%, the FFT approach may still be a reasonable approach to choose.

### 5.1 Future Work

The plausibility of the MISO radar system for applying STAP without increasing the number of receive channels leads to several areas of future interest. While the example waveform schemes discussed in this thesis illustrate many of the processing modifications to consider for a MISO radar system, other waveform schemes may lead to different considerations and result in preferable tradeoffs between imaging and STAP target detection. Phase-coded code division multiple access (CDMA) waveform designs would also be worthy of investigation. The adaptive weight vector training method would also be a future area of interest. As mentioned in Chapter 3, the clutter covariance rank may also be a consideration in the training method chosen for a MISO system. The clutter model of the FDMA system is worthy of investigation. Additionally, the minimum detectable velocity (MDV) of the MISO waveform schemes would also be useful for comparison purposes.

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